Welcome again to the **Foundation of applied machine learning**!

Problem 1: Python programming

This is a problem to check your coding skills! So **do not use** fancy modules and scripts!

Production of Fibonacci Sequence!

I have been assigned to write a function that takes an integer value n, where $n \in [0, 1, 2, 3, ...]$ and return the n^{th} value in Fibonacci sequence. Since, I am very lazy I just wrote the function **recursively** as you can see below. Another person, who was assigned to do the same thing, wrote the function with FOR LOOPS and claims his code is much faster!!

Algorithm 1: Recursive Fibonacci Function

```
def Fib_rec(n=0):
if n==0:
    return 1
elif n==1:
    return 1
else:
    return Fib_rec(n-1)+Fib_rec(n-2)
```

1. **Part 1:** Rewrite the function with for loop.
2. **Part 2:** Which function is actually faster? (Explain without running the codes)
3. **Part 3:** Write a code to time the **average** time for k times function call. A function that takes three arguments (function to time (Fib_rec), input of the function (n), number of runs(k)) and run the Fib_rec function k times for the input of n and returns the average time. ***Tip:*** *You can use the time module in the python:*

Algorithm 2: importing time modules

```
import time
# if you run this, the current time in (s) will be recorded in  x
x=time.time()
```

Algorithm 3: Timer Function

```
def timer(n, k, f=Fib_rec):
    <your code>
    return average_time
```

4. **Part 4:** Make a plot in which the x-axis is the value of the input function n and the y-axis is the average time (output of the previous function), for both recursive and non-recursive Fibonacci. (Both in the same plot; also use matplotlib package for making the plots)

Problem 2: Linear Algebra

Given the Matrix below answer the questions:

$$M = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$$

Part 1: Find the determinant, transpose, inverse(if exist) for M.

Part 2: Find the eigenvalues and eigenvectors for M.

Part 3: Find the Gradient if the $\nabla_A f(A)$ for the following:

$$A = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$$

$$f(A) = x_{11}^2 x_{22} x_{23} + x_{11} x_{12} x_{13} x_{31} - x_{33}^2 x_{32} x_{21}$$

Part 4: Find the Hessian Matrix for:

$$g(x, y, z) = x^3 y + yz \sin(x) + xy^2 z^5$$

Problem 3: Machine Learning

Part 1: Explain the difference between validation and test samples.

Part 2: Explain the difference between supervised and unsupervised learning algorithm.